# Cube & Rectangular Solids Solution

1. Cube has 6 faces. The surface area of a cube which has a side of 4cm is 6*4^2=6*16 cm^2.

   Now, when the cube is cut into the smaller cubes with side of 1cm we'll get 4*4*4=64 little cubes and each will have the surface area equal to 6*1^2=6 cm^2, so total surface are of these 64 little cubes will be 6*64 cm^2.

   6*64 is 4 times more than 6*16 which corresponds to 300% increase.

   Answer: D.

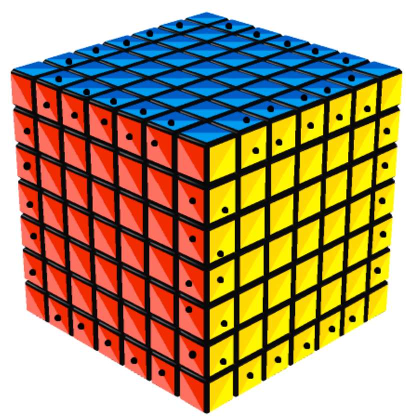
   Or: general formula for percent increase or decrease, (percent change): $Percent=\frac{Change}{Original}*100$

   So the percent increase will be: $Percent=\frac{Change}{Original}*100=\frac{6*64-6*16}{6*16}*100=300\%$.

   Answer: D.

2. Only the edge cubes marked with dots will have exactly two colored faces. 5 cubes per edge * 12 edges = 60 cubes.

   Answer: C



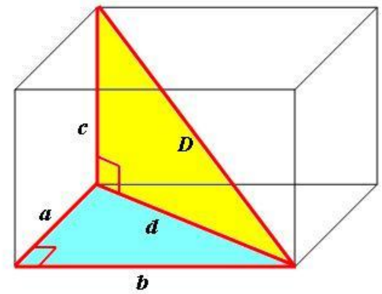
3. AC is the edge (the side) of a cube, suppose it equals to 1;
   AB is the diagonal of a face, hence is equals to $\sqrt{2}$, (either from 45-45-90 triangle properties or form Pythagorean theorem);
   BC is the diagonal of the cube itself and is equal to $\sqrt{1^2+1^2+1^2}=\sqrt{3}$;

   Ratio: $\frac{BC-AB}{AC}=\frac{\sqrt{3}-\sqrt{2}}{1}\approx 1.7-1.4=0.3$.

   Answer: C.

4. The longest distance will be the diagonal of a rectangular box. Look at the diagram in right.

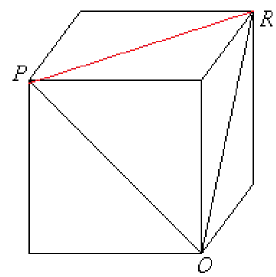
   Square of the diagonal of the face (base) is $d^2=a^2+b^2$ and the square of the diagonal of a rectangular box is $D^2=d^2+c^2=(a^2+b^2)+c^2$ --> $D=\sqrt{a^2+b^2+c^2}$.




   Applying this to our question, we get: $D=\sqrt{8^2+8^2+z^2}=\sqrt{128+z^2}$.

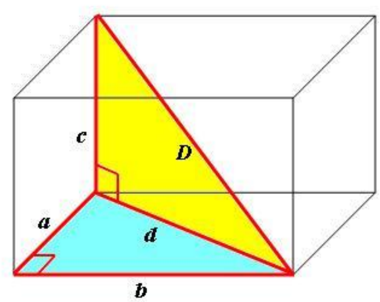
   Answer: E.

5. Note that triangle PQR is equilateral: it's made by the diagonals of the adjacent faces of the given cube (and as faces of a cube are squares its diagonals are equal). Thus angle BEG=60 degrees.




   Answer: C.

6. Square of the diagonal of the face (base) is $d^2=a^2+b^2$ and the square of the diagonal of a rectangular box is $D^2=d^2+c^2=(a^2+b^2)+c^2$ --> $D=\sqrt{a^2+b^2+c^2}$.




   Applying this to our question, we get: $D=\sqrt{10^2+10^2+5^2}=15$.

   Answer: A.

7. 1 million liter cube-shaped tank = 1,000,000/1,000 = 1,000 cubic meter tank --> the cube-shaped tank with a side of 10m (10^3=1,000).

   Surface area of this tank would be 6 (# of faces of a cube) * 10^2 (area of each face) = 600 m^2.

   # of 4*2=8m^2 sheet needed is 600/8=75.

   Answer: E.

8. Say the volume of the first cube is V=8, hence its side must be 2. Thus its surface area is 2^2*6=24.

   The surface are of the second cube will be 48, which means that the are of a face is 48/6=8. So, the side is $\sqrt{8}$. The volume = $(\sqrt{8})^3=16\sqrt{2}$.

   Now, plug V=8 into the answer choices and see which one yields $16\sqrt{2}$. Only option B fits.

   Answer: B.

9. Note that it should be mentioned that the side of the largest cube must be an integer (or dimensions should be 1 by 6, not 1 by 7).

   The wrapping paper has dimensions $1*3=3$ feet by $7*3=21$ feet. Thus, its area is $3*21=63$ square feet.

   Now, the surface area of a cube is $6a^2$, where $a$ is the length of a side (a cube has 6 faces and the area of each face is $a^2$).

   So, it must be true that $6a^2\leq 63$.

   If $a=4$, then $6a^2=96$, which means that we won't have enough paper for such cube.
   If $a=3$, then $6a^2=54$. The volume of such cube is $a^3=27$ cubic feet.

   Answer: D.

   If we won't limit the value of a side to integers only, then: $6a^2=63$ --> $a=\sqrt{10.5}$ --> $volume=a^3=(\sqrt{10.5})^3$.

10. Notice that, since the volume of each cube is 1 inch^3, then the volume of N cubes (the volume of the rectangular box) is N inch^3. For example if there are 10 cubes, then the volume of the rectangular box (total volume of 10 cubes) is 10 inch^3. Next, we are told that the length, the width and the height of the rectangular box is longer than 1 inch and there are no gaps when all cubes are put in the box, so the length, the width and the height of the rectangular box are integers more than one: 2, 3, 4, ... Thus each dimension of the rectangular box must have at least one prime in it, so the volume (length*width*height) must be the product of at least 3 primes (not necessarily distinct primes).

    (1) 56 < N < 63. N could be 57, 58, 59, 60, 61, or 62. Analyze each case:

    57=3*19 --> just two primes. Discard.
    58=2*29 --> just two primes. Discard.
    59 --> prime itself. Discard.
    60=2^2*3*5 --> the product of 4 primes. OK. For example, the the length, the width and the height of the cube cold be 2, by 6, by 5.

61 --> prime itself. Discard.
62=2*31 --> just two primes. Discard.

As we can see, N can only be 60. Sufficient.

(2) N is a multiple of 3. Multiple values of N are possible so that it to be a multiple of 3 AND the product of at least 3 primes, for example 27 or 60. Not sufficient.

Answer: A.

11. Since between 80 and 85 percent of the cube's volume is below the surface of the water and between 12 and 16 cubic centimeters of the cube's volume is above the surface of the water, then 85-80=5% or 16-12=4 cubic centimeters of the volume is sometimes below and sometimes above the surface of the water. V*5%=4 cubic centimeters --> V=80 --> $side=\sqrt[3]{80}=2\sqrt[3]{10}\approx 4.2$.

    Answer: A.

12. We have 160+56=216=6^3 big cube.

    Non exposed will be 4^3 little cubes, which will be "inside" the big cube, so exposed will be 216-4^3=152 little cubes out of which there will be 152-56=96 colored cube (as colored cubes are minimized on the surface).

    Now surface area of big cube is 6*36=216 (exposed area) out of which the area of 96 is colored, so the percentage of exposed area THATis coloured is --> $\frac{96}{216}\approx(0.4444)=44.44\%$.

    Answer: B.

13. Take the diagonals of adjacent sides of a cube, so that they make an angle.

    Join the ends of these diagonals to get the triangle.

    You'll see that the line segment joining the ends of these diagonals will also be the diagonal of a side of a cube.

    Hence we have the triangle all 3 sides of which are diagonals of a side of a cube. All diagonals are equal, which means that we have equilateral triangle. Any angle in equilateral triangle is 60 degrees. Answer is C.

14. The cube is composed of 10^3 little cubes. Out of them 8^3 non-exposed central cubes won't be colored at all.

    Thus the probability is P(at least one red face) = 1 - P(no red face) = 1 - 8^3/10^3= 488/10^3.

    Answer: B.

15. As the cube consists of 125=5^3 identical small cubes then its sides are built with 5 small cubes. The sides of the inner cube are built with 3 small cube (5 - 2 small edge cubes), thus there are 125-3^3=98 cubes exposed.

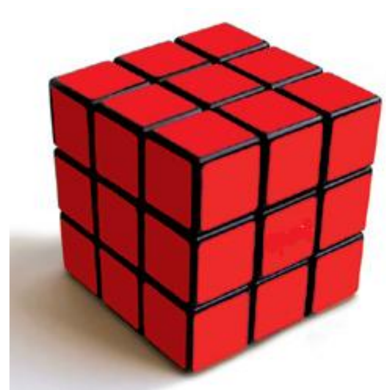
    Answer: C.

16. The side of the cube is N. The area is $N^3 = 64$ so $N = 4$, there are 4 cubes on each side To make this cube "one cube longer" we have to add one cube at both ends of a side $1+4+1=6$ , the new cube will have an area of $6^3 = 216$ cubes.

    The difference in the areas will be the number of cubes we've added: $216-64=152$

    Answer is C.

17. Say n=3.

    So, we would have that the large cube is cut into 3^3=27 smaller cubes:

    Out of them only the central little cube won't be painted red at all and the remaining 26 will have at least one red face. Now, plug n=3 and see which one of the options will yield 26. Only B works: 6n^2 – 12n + 8 = 54 - 36 + 8 = 26.

    Answer: B.

18. Let's try this question using only the change in volume. Since original volume = final volume, Decrease in volume = Increase in volume

    You decrease volume by chopping off 9 inches of the height. Decrease = 9*w*l
    You increase the volume now by adding an inch to the width and length. The height remains h-9.
    Increase = 1*(l+1)*(h-9) + 1*w*(h-9) = (h-9)*(l+w+1) (make a rectangle and increase its width and length by 1 to see how area changes. This tells you how volume changes by just considering the height as well)

    So 9*w*l = (h-9)*(l+w+1)
    Given that (h-9) = 4w and l = w, substitute both in the equation to get
    9*w*w = 4w*(2w+1)
    Cancel w from both sides and get w = 4 = l
    h-9 = 4w so h = 16+9 = 25

    Volume = wlh = 4*4*25 = 400

19. Inner volume of the box is $xyz$ cubic centimeters.

    Now, since each of the six sides of the box is 1 centimeter thick, then outer dimensions are $x+2$ by $y+2$ by $z+2$ centimeters. Therefore, the volume of the box with aluminium is $(x+2)(y+2)(z+2)$ cubic centimeters.

    The volume of the aluminium is the difference of these two: $(x+2)(y+2)(z+2)-xyz=2(xy+xz+yz+2x+2y+2z+4)$.

    Answer: E.

    OR:

    Plug numbers: say $x=y=z=1$, then the inner volume is 1 cubic centimeters and the volume of the whole box is $(1+2)(1+2)(1+2)=27$ cubic centimeters. The volume of the aluminium is 27-1=26 cubic centimeters.

    Now, plug $x=y=z=1$ and see which options yields 26: only answer choice E fits.

    Answer: E.

    P.S. For plug-in method it might happen that for some particular numbers more than one option may give "correct" answer. In this case just pick some other numbers and check again these "correct" options only.

20. The options are well spread so we can approximate.

    Changing the length by 1 cm results in change of the volume by 1*200*300 = 60,000 cubic centimeters;
    Changing the width by 1 cm results in change of the volume by 200*1*300 = 60,000 cubic centimeters;
    Changing the height by 1 cm results in change of the volume by 200*200*1 = 40,000 cubic centimeters.

    So, approximate maximum possible difference is 60,000 + 60,000 + 40,000 = 160,000 cubic centimeters.

    Answer: C.

21. We have a rectangular solid to pack cubes, which has the volume of 64 cubic inches, (side=4).

    (1) The volume of the container is 1024 cubic inches. The volume of a rectangular solid is not enough to get the dimensions. If the dimensions are 1*1*1024, then you wouldn't fit any box in the tanks but if the dimensions are 4*4*64, then you would fit 16 boxes in the tank. Not sufficient.

(2) The opening of the container is 16 inches by 16 inches. We need the third dimension to answer the question. Not sufficient.

(1)+(2) From above the dimensions of the tank are 16*16*4. Sufficient.

Answer: C.

22. From the stem we have: width*depth = 60;
From (1) we have that: width*height = 60.

Thus: width*depth = width*height, which gives that depth = height. Now, if depth = height = 1 and width = 60, then the block is NOT a cube but if depth = height = width = $\sqrt{60}$ , then the block is a cube.

The same logic applies to (2): we get that width = height.

For (1)+(2) we have that depth = height and width = height, therefore depth = height = width, which means that the block is a cube.

Answer: C.

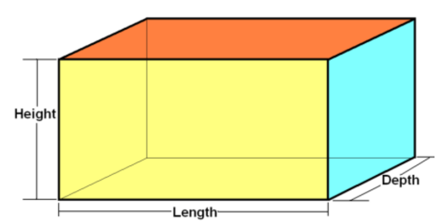
23. (1) The sum of the areas of the faces of the cube is 54. A cube has 6 faces and the area of each is a^2, where a is the length of a side. Thus we have that $6a^2=54$ --> $a=3$ --> $volume=a^3=27$. Sufficient.

(2) The greatest possible distance between two points on the cube is $3\sqrt{3}$ . The implies that the diagonal of the cube is $3\sqrt{3}$ --> $diagonal=a^2+a^2+a^2=(3\sqrt{3})^2$ --> $a=3$ --> $volume=a^3=27$. Sufficient.

Answer: D.

24. In a rectangular solid, all angles are right angles, and opposite faces are equal, so rectangular solid can have maximum 3 different areas of its faces, on the diagram: yellow, green and red faces can have different areas. I say at max, as for example rectangular solid can be a cube and in this case it'll have all faces equal, also it's possible to have only 2 different areas of the faces, for example when the base is square and the height does not equals to the side of this square.

Volume of rectangular solid is Volume=Length*Height*Depth.




BACK TO THE ORIGINAL QUESTION:

What is the volume of a certain rectangular solid?

(1) Two adjacent faces of the solid have areas 15 and 24, respectively --> let the two adjacent faces be blue and yellow faces on the diagram --> $blue=d*h=15$ and $yellow=l*h=24$ --> we have 2 equations with 3 unknowns, not sufficient to calculate the value of each or the product of the unknowns ($V=l*h*d$).

To elaborate more:
If $blue=d*h=15*1=15$ and $yellow=l*h=24*1=24$ then $V=l*h*d=24*1*15=360$;
If $blue=d*h=5*3=15$ and $yellow=l*h=8*3=24$ then $V=l*h*d=8*3*5=90$.

Two different answers, hence not sufficient.

(2) Each of two opposite faces of the solid has area 40 --> just gives the areas of two opposite faces, so clearly insufficient.

(1)+(2) From (1): $blue=d*h=15$, $yellow=l*h=24$ and from (2) each of two opposite faces of the solid has area 40, so it must be the red one: $red=d*l=40$ --> here we have 3 distinct linear equations with 3 unknowns hence we can find the values of each and thus can calculate $V=l*h*d$. Sufficient.

To show how it can be done: multiply these 3 equations --> $l^2*h^2*d^2=(l*h*d)^2=15*24*40=24^2*5^2$ --> $V=l*h*d=24*5=120$.

Answer: C.

25. Let the side of the cube be $x$, then the volume will be $volume=x^3$

(1) The surface area of the cube is 600 square inches --> surface area of a cube equals to {area of a face} * {# of faces} --> $surface\ area=x^2*6=600$, --> $x=10$--> $volume=x^3=1,000$. Sufficient.

(2) The length of diagonal AB is $10\sqrt{3}$ inches --> diagonal of a cube equals to $Diagonal=\sqrt{x^2+x^2+x^2}=x\sqrt{3}=10\sqrt{3}$(x^2+x^2=square of a diagonal of a face and square of a diagonal of a face plus square of a side equals to diagonal of a cube) --> $x=10$--> $volume=x^3=1,000$. Sufficient.

Answer: D.